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RELIABILITY AND INVERSE RELIABILITY IN EARTHQUAKE ENGINEERING DESIGN

Foschi, Ricardo O.[1] and Li, Hong[2]

ABSTRACT

This paper discusses the application of forward and of inverse reliability procedures in the context of earthquake engineering. It is argued that design guidelines for specific limit states must be developed taking into account the full nonlinear soil/structural interaction response, the uncertainty in the ground motion and in the structural parameters, with an acceptable risk of non-performance defined for the particular event or on an annual basis. The procedure is illustrated with the design of a single pile foundation, in which the mass M carried by the pile at the cap must be designed to achieve a given reliability against a limiting lateral pile-head displacement during an earthquake. The example illustrates reliability assessment (forward reliability) and reliability-based design (the inverse problem of estimation of design parameters given a reliability level).

INTRODUCTION

Given the high degree of uncertainty in the earthquake ground motion, coupled with other uncertainties in structural behaviour and modelling, design problems in earthquake engineering should be at the forefront of applications of probabilistic methods or reliability-based design. This would involve several aspects: 1) appropriate definition of the limit states of interest (collapse, serviceability or damage states), 2) quantification of the intervening random variables and processes, 3) analysis of the nonlinear dynamic response of the system soil/structure to the seismic excitation, including realistic quantification of hysteretic behaviour, and 4) the integration of the dynamic response with evaluation of structural reliability, or the probability that the performance standard set for the limit states will be met within the time span of interest. The results of such an evaluation process can be used in two ways: 1) by themselves, in proper reliability-based design, or 2) to calibrate "simplified" code design procedures which, when applied, would approximately ensure a desired reliability target.

For most types of structures, except those of major importance, code specifications follow an approach whereby elastic stresses and deformations, calculated with linear response analyses, are then modified by a series of coefficients meant to account for the influence of soil conditions, dynamic characteristics of the structure and the "ductile" behaviour built into the design. The inertia forces are calculated for a "design hazard" (e.g., a localized design acceleration defined at a level to be exceeded with a prescribed probability on an annual basis). Response spectra may also be constructed to represent situations with a certain confidence that they will not be exceeded. This method should not be seen as a "probabilistic" approach in the sense that it provides an evaluation of the reliability achieved. In fact, even if the code procedures were calibrated to a target reliability for a number of design situations, it is unlikely that they would maintain the same target for all different situations.

However, proper reliability evaluation poses challenging problems, particularly modelling the nonlinearities in the dynamic response as a function of design parameters. One such important nonlinearity is the structural response to time-varying excitation as it is influenced by the hysteretic characteristic of the relationship between cyclically applied forces and the resulting displacements. For example, Figure 1 shows a pile supporting a mass M and undergoing an

[1] Professor, Dept. of Civil Engineering, University of British Columbia, Vancouver, B.C. V6T 1Z4
[2] Ph.D. Graduate Student, Dept. of Civil Engineering, University of British Columbia, Vancouver, B.C. V6T 1Z4

earthquake-induced acceleration $a_G(t)$. The relationship between the horizontal displacement Δ of the pile cap and the shear V is typically in the form of a hysteresis loop, representing the nonlinear response resulting from the elasto-plastic properties of the pile and the nonlinear interaction between the surrounding soil and the pile.

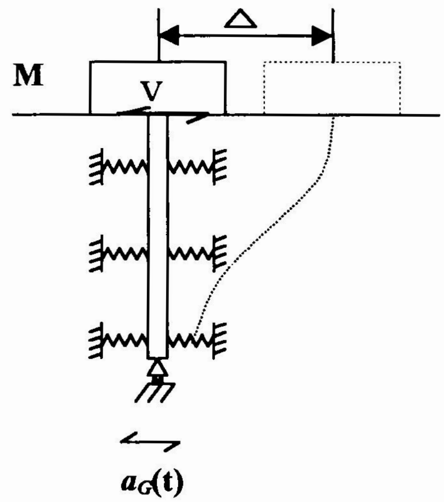

Figure 1. Pile configuration

In addition, the formation of gaps between the pile and the supporting medium, which result from the absence of tensile stresses at the interface soil/pile, and which introduces structural looseness, gives a "pinched" characteristic to the hysteretic loop. Gap development and P-Δ amplification effects must be accounted in the analysis, particularly for limit states associated with excessive deformations or damage levels. Given that the area enclosed by the loop is a measurement of the energy dissipated during the displacement history, it is obvious that hysteretic properties play a central role in the analysis of the dynamic response of structures to the ground motion. It is essential, therefore, to include in the dynamic analysis a reasonably accurate representation of hysteresis.

Empirical hysteresis models are usually constructed by specifying a set of rules for loading and unloading, involving a set of parameters which are calibrated to an observed experimental response. This approach ranges from coarser models using straight segments between changes in displacement direction, to more sophisticated techniques. Among the latter, the procedure developed by Baber, Noori and Wen (1981, 1985) and modified slightly by Foliente (1995), is noted. In it, the total force is separated into a linear and a hysteretic component and a first-order differential equation for the hysteretic force is integrated as a function of the displacement, as this progresses over time. The differential equation contains 13 parameters which must be calibrated to tests. The parameters provide sufficient flexibility to account for yielding, pinching, strength and stiffness degradation. When such empirical models are fitted to a known cyclic response, there is of course no assurance that they will properly model the response for any other excitation history.

Another approach is to *calculate* the hysteresis loop by solving, at each time step, the nonlinear problem of the structural response to the imposed history. This response can be calculated from basic stress/strain information on the structural member, and knowing the nonlinear compressive behaviour of the medium surrounding that member. The main advantage of this formulation is that, starting from basic properties, it will automatically adjust to any imposed history. Although calculating the hysteresis loop has been commonly applied in geotechnical engineering, applications in structural engineering have relied more on the empirical models previously described. This paper presents a study of the reliability evaluation for the pile in Figure 1, when the hysteretic response is calculated at every step of the time history. The limit state or performance criterion considered is one of limiting the maximum displacement Δ occurring during the earthquake, a limit which may have been imposed because of allowable damage levels. The analysis model is briefly presented, followed by the reliability evaluation for a given mass M. Inversely, the mass M meeting the reliability standard can be calculated. Details on the inverse approach are presented in separate papers (Li and Foschi, 1998, 1999), including an analysis of multiple solutions for the same target reliability.

THE PILE MODEL

The approach is based on the response of an elasto-plastic beam on a nonlinear foundation, with an interface which only acts in compression. A beam finite element formulation can be used for the structural member, using higher-order interpolating functions to minimize the number of required elements. This approach, not using finite elements and restricting the analysis to elastic piles, has already been discussed in the response of pile foundations (Finn et al. 1992, Gohl 1991). In geotechnical engineering, the response of the soil to compressive forces is commonly called the p-y curve. There has also been work done on calculating the hysteretic response using plasticity theory for the surrounding medium, rather than p-y curves (Finn et al. 1997, Cai et al. 1997). Here, the pile length L is divided into elements within which the lateral displacement w and the axial displacement u are expressed, respectively, as fifth and third degree polynomials in x. These displacements are referred to the center of gravity of the pile cross-section. In order to take into account deflection amplifications due to the P-Δ effect of the axial load, the strain ε at a distance y from the center of gravity is expressed as

$$\varepsilon = \frac{\partial u}{\partial x} - y\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \tag{1}$$

The stress $\sigma(\varepsilon)$ in the member is assumed to obey an elasto-plastic constitutive relation, with either strain hardening or softening. The reaction forces p from the soil medium, per unit length, are assumed to be a function $p(w)$ of the displacement w only when p is a compressive force. Otherwise, $p = 0$. The principle of virtual work is then used to formulate the linear and the nonlinear terms associated with the stiffness of the system, and the consistent mass matrix. The resulting equations of motion are integrated in the time domain using a constant average acceleration routine and, within each time step, iterations are carried out by the Newton-Raphson technique, taking into account the tangent stiffness matrix, until dynamic equilibrium is achieved. Complete details on the model's development are shown elsewhere (Foschi 1998). The model can be used to calculate the shear force V required at the pile cap to produce a lateral displacement Δ . In the process, the pile shape is obtained at different depths, along with the magnitude of the gaps developing between the pile and the soil medium. Because of these gaps, the relationship between the force V and the displacement Δ is a pinched hysteresis loop. One can also calculate the displacement produced by an input force history. In the case of an earthquake input, the force history is proportional to the inertia force given by the mass M and the ground acceleration $a(t)$.

The pile studied was a steel tube with a length $L = 30$ *m*, outside diameter of *356 mm*, and a wall thickness of *10 mm*. The modulus of elasticity was $E = 200$ *GPa* and, assuming a elastic-perfectly plastic response, the yield stress was $\sigma_Y = 0.25$ *GPa* . The response from the soil was characterized by the Yan-Byrne relationship (Yan et al. 1992), using a soil specific weight $\gamma = 20.0\ kN/m^3$ and a relative density $D_R = 75\%$. It is noted that, in the Yan-Byrne model, the soil properties change with depth, as a function of the overburden. Figures 2 and 3 show the calculated pile hysteretic response to two sinusoidal inputs $\Delta(t)$. Each includes five complete periods with a total duration of five minutes. In Figure 2, the amplitude increases from *0 to 0.25 m*. In Figure 3, the amplitude is constant and equal to 0.05 m. The loop in Figure 3 reflects lack of yielding in the pile, with the hysteresis reflecting only the nonlinear soil behavior. On the other hand, the loop in Figure 2 corresponds to displacements sufficiently large to result in plastic deformation in the pile, with additional energy dissipation and a fuller, more rounded loop.

RELIABILITY EVALUATION

Let us now consider the reliability when the mass $M = 45$ *Tonnes*. What is desired is the probability with which different pile-cap displacements will be exceeded during the earthquake. The earthquake peak ground acceleration a_G was assumed to have a Lognormal distribution with a mean of *0.103g* and a coefficient of variation of *0.6*. Assuming a mean event arrival rate $\nu = 0.05$, the corresponding *design* peak ground acceleration (with a return period of 475 years) equals *0.23g*. The earthquake time history was the historical 1992 Landers (California) event, Joshua Tree Station, scaled to a peak acceleration of 1 m/sec^2. First, only the peak ground acceleration a_G was considered a random variable. All pile and soil properties were assumed deterministic and equal to their mean values. It is then simple to construct the curves shown in Figure 4, representing the relationship between the mass M and the maximum pile-cap displacement Δ at a given reliability level β. These curves represent reliability contours, and can be obtained by executing the deterministic dynamic analysis with different masses for specified peak ground accelerations. Since there is only one random variable, the value of a_G corresponding, for example, to $\beta = 2.5$ is $a_G = 0.35g$. Similarly, $a_G = 0.47g$ and $a_G = 0.62g$ correspond, respectively, to $\beta = 3.0$ and $\beta = 3.5$.

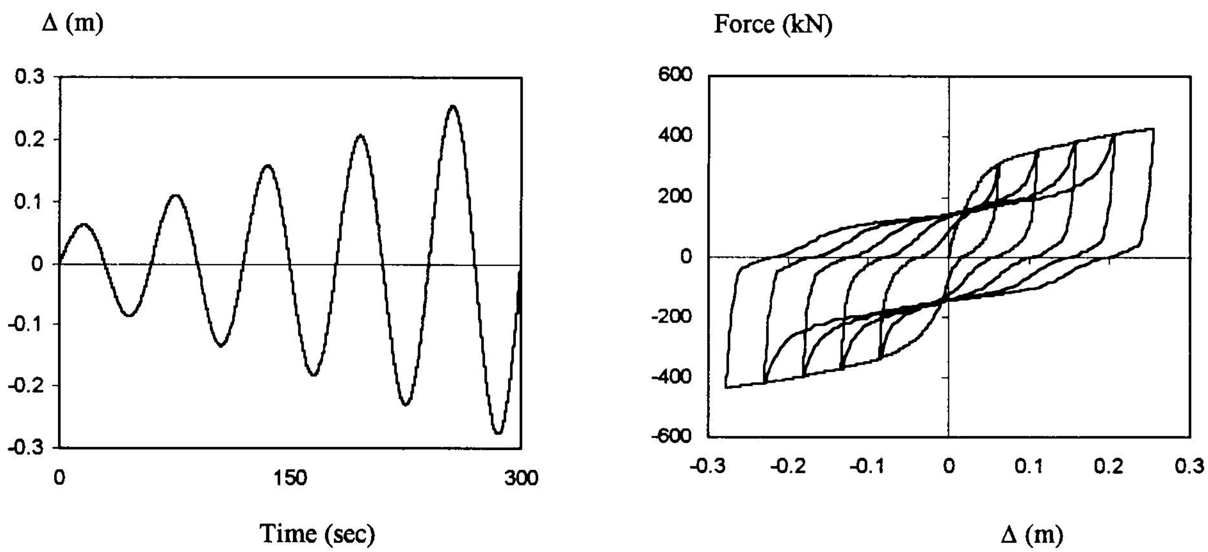

Figure 2. Sinusoidal Displacement and Calculated Hysteresis

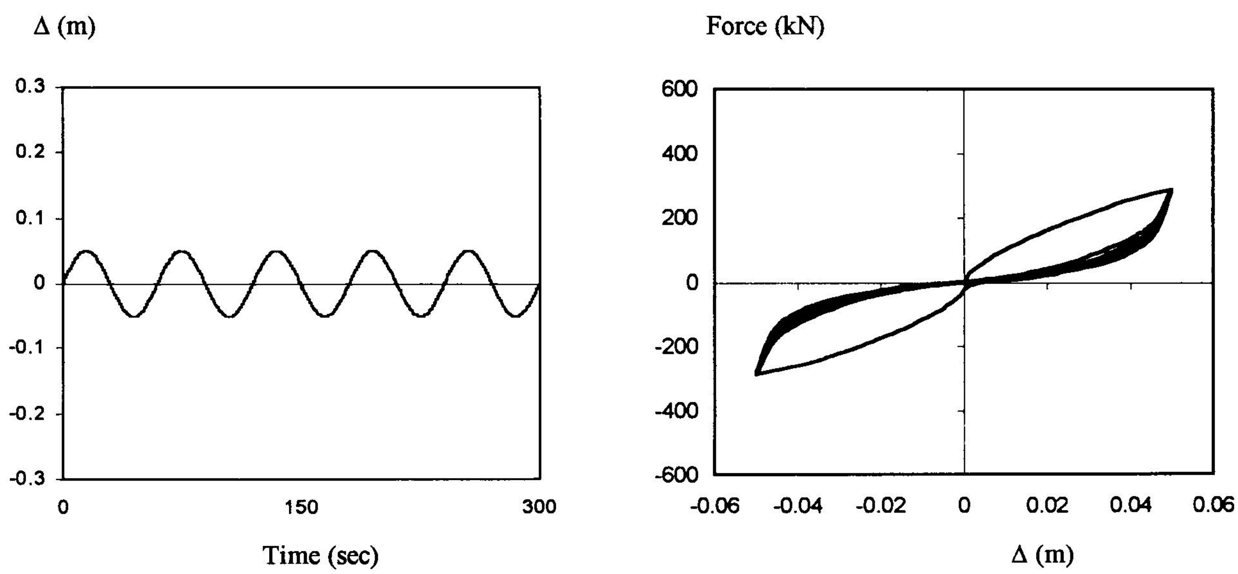

Figure 3. Sinusoidal Displacement and Calculated Hysteresis

Figure 4 can be used to estimate, at any mass, the reliability level associated with a given displacement Δ. In particular, at $M = 45$ *Tonnes*, $\Delta = 0.112m$, $\Delta = 0.139m$, and $\Delta = 0.224m$ correspond, respectively, to reliability levels $\beta = 2.5$, $\beta = 3.0$ and $\beta = 3.5$. These results are shown in Table 1.

The inverse reliability problem of finding M at a given β is also simplified since only one random variable is used. Figure 4 shows that, for a limit $\Delta = 0.10m$ and $\beta = 3.5$, the mass should be approximately M = 28.3 Tonnes. The estimate for this mass can be updated by releasing more variables from deterministic to random. The estimate will not change substantially when the peak ground acceleration is, from a reliability viewpoint, the most important one. However, when several random variables are considered, the calculation of the reliability has to be done by special software. In this case, the package RELAN (Foschi et al., 1998) was used with the performance function G

$$G = \Delta_{LIM} - \Delta \quad (2)$$

where $\Delta_{LIM} = 0.10m$ is the limit deflection and Δ is the maximum deflection recorded during the earthquake with a mass M. The reliability estimate reduces from $\beta = 3.50$ to $\beta = 3.39$, as shown in Table 2, which also contains the most likely failure combination of the three random variables considered.

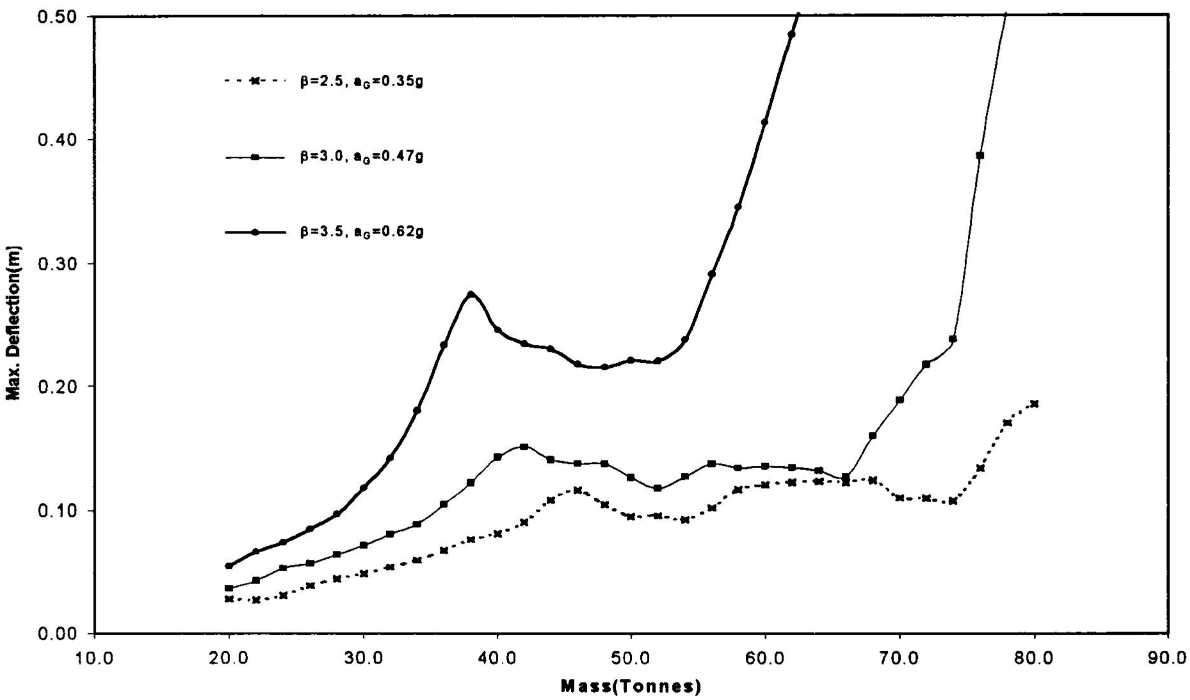

Figure 4. Relationship Between Mass and Maximum Deflection at Specified Reliability Levels β.

Table 1. Reliability corresponding to different deflection limits, @ $M = 45$ Tonnes

Deflection Δ_{LIM} (m)	Reliability Index β	Probability $(\Delta > \Delta_{LIM})$
0.112	*2.5*	$0.621 x 10^{-2}$
0.139	*3.0*	$0.135 x 10^{-2}$
0.224	*3.5*	$0.233 x 10^{-3}$

Table 2. Mass M and Most Likely Failure Combination, @ Δ_{LIM} = 0.10m.

Mass (Tonnes)	Reliability Index β	Probability $(\Delta > \Delta_{LIM})$	σ_Y (Gpa)	D_R (%)	a_G (g)
28.3	*3.39*	$0.347x10^{-3}$	*0.246*	*70.95*	*0.575*

CONCLUSIONS

The usefulness of reliability approaches in structural/geotechnical engineering lies not so much in being able to estimate the reliability of a system, but in also designing the system to a target reliability. All problems, particularly the calibration of code procedures, should use such approaches. They have been illustrated with the design of a single pile foundation under earthquake loading, implementing the required nonlinear dynamic analysis within the reliability evaluation. Although the example presented used only one earthquake accelerogram, the procedure is equally applicable to a combination of randomly generated earthquakes, adding only more random variables (e.g., random phases) to represent the ground motion.

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